Eight Mathematical Practices–Cubed!

Understanding Ohio's 2017 Revised Math Standards

Standard 1: Make sense of problems and persevere in solving them.

Standard 2: Reason abstractly and quantitatively.

Standard 3: Construct viable arguments and critique the reasoning of others.

Standard 4: Model with mathematics.

Standard 5: Use appropriate tools strategically.

Standard 6: Attend to precision.

Standard 7: Look for and make use of structure.

Standard 8: Look for and express regularity in repeated reasoning.

Exploring What's New in Ohio's 2017 Revised Math Standards

- No Revisions to Math Practice Standards.
- Minor Revisions to Grade Level and/or Content Standards.
- Revisions Clarify and/or Lighten Required Content.

Drill-Down Some Old/New Comparisons…

Clarify Kindergarten Counting:

K-12 Mathematics Standards Comparison 1 / 58

Key:
Red shows added words.
Purple shows a footnote/course focus.

K-12 Mathematics Standards Comparison

Domain/ Conceptual Category	Standard	Original Standard	New Standard
Counting and Cardinality	K.CC.1	**Know number names and the count sequence.** **K.CC.1** Count to 100 by ones and by tens.	No change
Counting and Cardinality	K.CC.2	**Know number names and the count sequence.** **K.CC.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	**Know number names and the count sequence.** **K.CC.2** Count forward within 100 beginning from any given number other than 1.
Counting and Cardinality	K.CC.3	**Know number names and the count sequence.** **K.CC.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	**Know number names and the count sequence.** **K.CC.3** Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
Counting and Cardinality	K.CC.4	**Count to tell the number of objects.** **K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality.	**Count to tell the number of objects.** **K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies.

Lighten Adding 5th Grade Fractions:

Key:
Red shows added words.
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K-12 Mathematics Standards Comparison

Domain/ Conceptual Category	Standard	Original Standard	New Standard
Numbers and Operations–Fractions	5.NF.1	**Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	**Use equivalent fractions as a strategy to add and subtract fractions (Fractions need not be simplified.)** **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show 2/3 + 5/4 = 8/12 + 15/12 = 23/12. In general, a/b + c/d = (a/b x d/d) + (c/d x b/b) = (ad + bc)/bd.*
Numbers and Operations–Fractions	5.NF.2	**Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole,	No change

Lighten Solving High School Quadratics:

K-12 Mathematics Standards Comparison 42 / 58

Key:
Red shows added words.
Purple shows a footnote/course focus.

K-12 Mathematics Standards Comparison

Domain/ Conceptual Category	Standard	Original Standard	New Standard
Algebra	A.REI.4	**Solve equations and inequalities in one variable.** **A.REI.4** Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **b.** Solve quadratic equations by inspection e.g., for $x^2 = 49$, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	**Solve equations and inequalities in one variable.** **A.REI.4** Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (+) **c.** Derive the quadratic formula using the method of completing the square.
Algebra	A.REI.5	**Solve systems of equations.** **A.REI.5** Prove that, given a system of two	**Solve systems of equations.** **A.REI.5** Verify that, given a system of two equations in

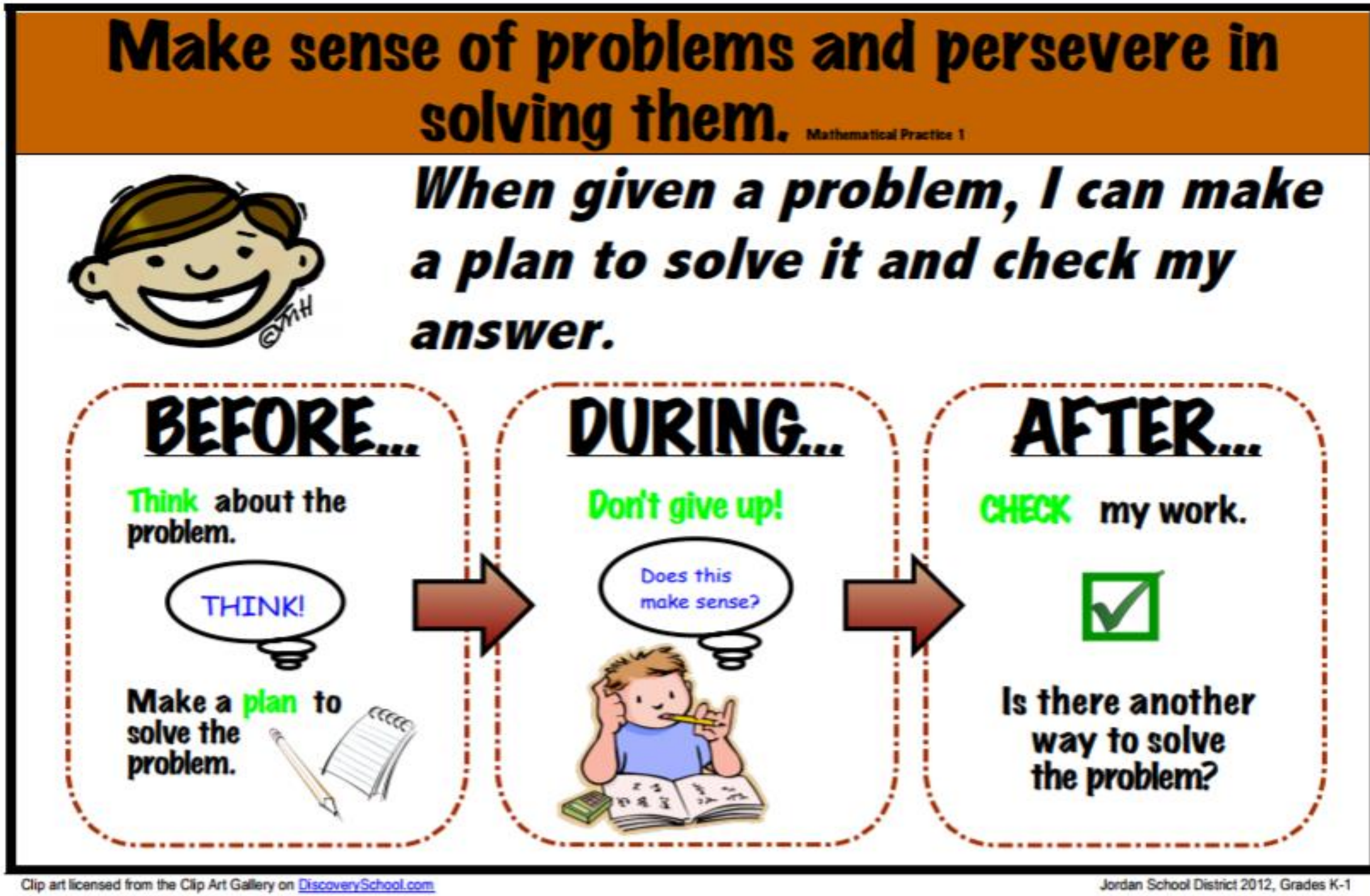
Ohio 2017 Math Practice Standard #1

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Jordan District Posters

Seeing Math Practice #1 from a K-1 Student Perspective



Jordan District Posters

Seeing Math Practice #1 from a Grade 2-3 Student Perspective

Make sense of problems and persevere in solving them.

Mathematical Practice 1

When given a problem, I can make a plan, carry out my plan, and check my answer.

BEFORE...

Think about the problem.

Ask myself, "Which strategy will I use?"

Make a plan to solve the problem.

DURING...

Stick to it!

Ask myself, "Does this make sense?"

Change my plan if it isn't working out.

AFTER...

CHECK my work.

☑

Ask myself, "Is there another way to solve the problem?"

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Jordan School District 2012, Grades 2-3

Seeing All Eight Ohio Math Practices
From Euclid to Euler, Edison, Einstein etc.

Solve Tenaciously 1: Make sense of problems and persevere in solving them;

Quantify Contextually 2: Reason abstractly and quantitatively;

Argue Logically: Construct viable arguments and critique the reasoning of others;

Model Realistically 4: Model with mathematics;

Tool Incisively 5: Use appropriate tools strategically;

Communicate Precisely 6: Attend to precision;

Structure Hierarchically 7: Look for and make use of structure;

Iterate Recursively 8: Look for and express regularity in repeated reasoning.

Condensing the Practice Statement

Such As: ***1. Make sense of problems and persevere in solving them.***

Step One. *Leave the boilerplate, take the cannoli!*

"~~Mathematically proficient students start by~~ **explain**~~ing to themselves~~ **the meaning of a problem and look**~~ing~~ **for entry points to its solution.** ~~They~~ **analyze givens, constraints, relationships, and goals** etc. etc. etc."

Step Two. *Leave age references, take adult practices!*

"~~Younger students might rely on~~ use concrete objects or pictures to ~~help~~ conceptualize and solve a problem."

Refining an **Essential Practice:** One Imperative, Two Sentences

1. Make sense of problems and persevere in solving them. Explain to yourself the meaning of a problem and look for entry points to its solution. **Analyze the givens, constraints, relationships and goals.** Conjecture the form and meaning of a solution before jumping in. Consider analogous problems, **special cases and simpler versions** of the problem to gain insights into the solution. **Monitor** and evaluate **your progress**—change course if necessary. Apply correspondences between equations, verbal descriptions and tables. Draw diagrams of important features. Seek regularities and trends. Use concrete objects or pictures to conceptualize and solve the problem. **Check your answer by another method**—continually ask, "Does it make sense?" Understand approaches of others to the problem; identify correspondences among multiple approaches.

1. Solve Tenaciously! Analyze your problem's givens, constraints, relationships, goals, special cases and simpler versions. Monitor your progress and check your answers by another method.

Thumbnail Standards for the 8 Math Practices

Solve! Analyze your problem's givens, constraints, relationships, goals, special cases simpler versions. Monitor progress and check your answers by another method.

Quantify! Manipulate symbols as if they have a life of their own, but pause to probe referents for the symbols. Consider the units involved.

Argue! Use established results and logical statements to explore conjectures. Communicate your conclusions and justify them to others.

Model! Apply mathematics to everyday life and society. Use functions, diagrams and formulas to model real quantities and draw practical conclusions.

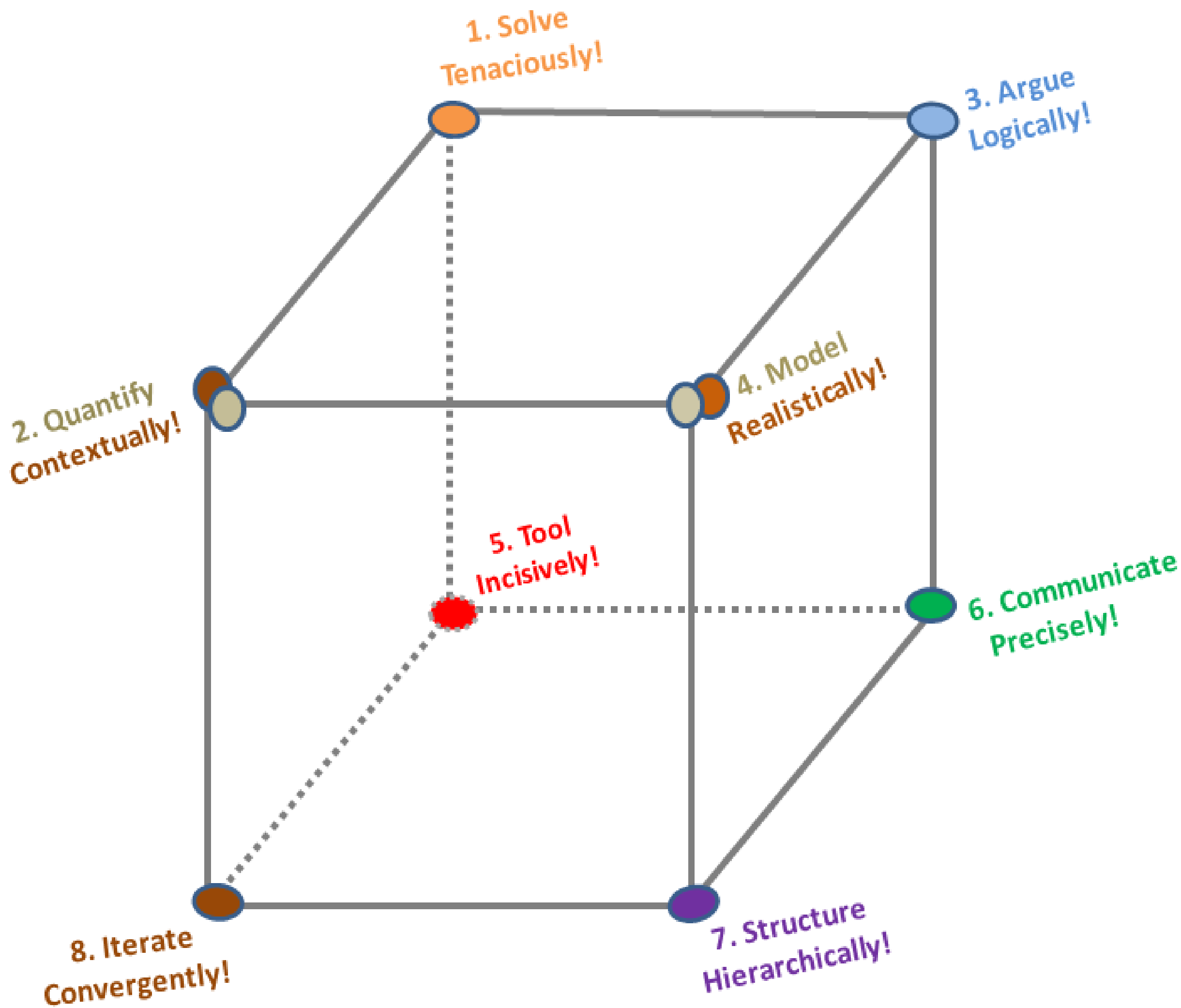
Tool! Consider pencil and paper, concrete models, computers, spreadsheets and calculators. Use external websites to pose and explore problems.

Communicate! Formulate explanations and examine claims using stated symbols and explicit definitions. Specify units of measure and use appropriate numerical precision.

Structure! Discern significant lines in a geometric figure and draw auxiliary lines. View an algebraic expression as one object or a composition of significant objects.

Iterate! When your calculations repeat, seek general methods and cumulative shortcuts. Investigate reasonableness of your problem-solving process while attending to its details.

Eight Mathematical Practices—Cubed!



Math Content Practices

Rational Discourse Practices

1. Solve! Analyze givens and check answers.

2. Quantify! Manipulate symbols and consider units.

3. Argue! Use logical statements; justify them.

4. Model! Apply math to life and draw practical conclusions.

5. Tool! Consider computers and websites.

6. Communicate! Formulate explanations.

7. Structure! Discern composition of objects.

8. Iterate! When calculations repeat, seek shortcut methods.

Back Face of the Praxis Cube—
“Rational Discourse ‘R Us!”

- **1. Persistence** produces timely inventions and insights.
- **3. Reason** justifies irrefutable conclusions.
- **5. Tools** magnify and focus human effort.
- **6. Precise Expression** guarantees compelling communication.

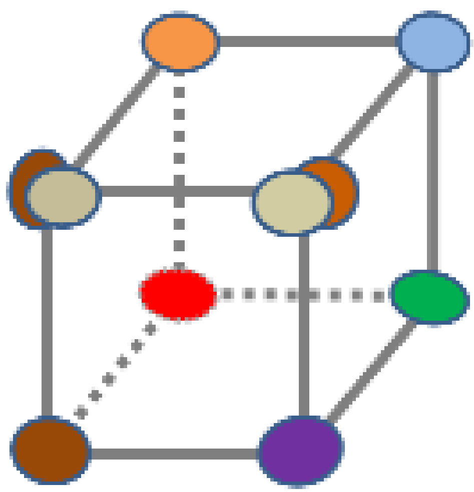
Persistence, Reason, Tool Use, and Clarity
are general habits of mind and action.

Front Face of the Praxis Cube—
Four Patterns of Mathematical Thought

- **2. Quantifying** is the ‘word-problem’ pattern:
  let x equal; solve-for-x; restate-physical-units, etc.
- **4. Modeling** is the social-law/natural-law pattern:
  `   bell-shaped distributions, laws of motion, etc.
- **7. Structuring** is the finite-math pattern:
  graph theory, prime number theory, and so forth.
- **8. Iterating** is the continuous-math pattern:
  convergent infinite series, calculus and so forth.

Quantifying, Modeling, Structuring and Iterating
are <u>mathematical</u> patterns of investigation.

What the Colors Mean!



Top Face

1. Persistence = Orange
2. Quantifying = Gray/Brown
3, Reason = Blue
4. Modeling = Silver/Gold

Bottom Face

5. Tool Use = Red
6. Clarity = Green
7. Structuring = Purple
8. Iterating = Brown

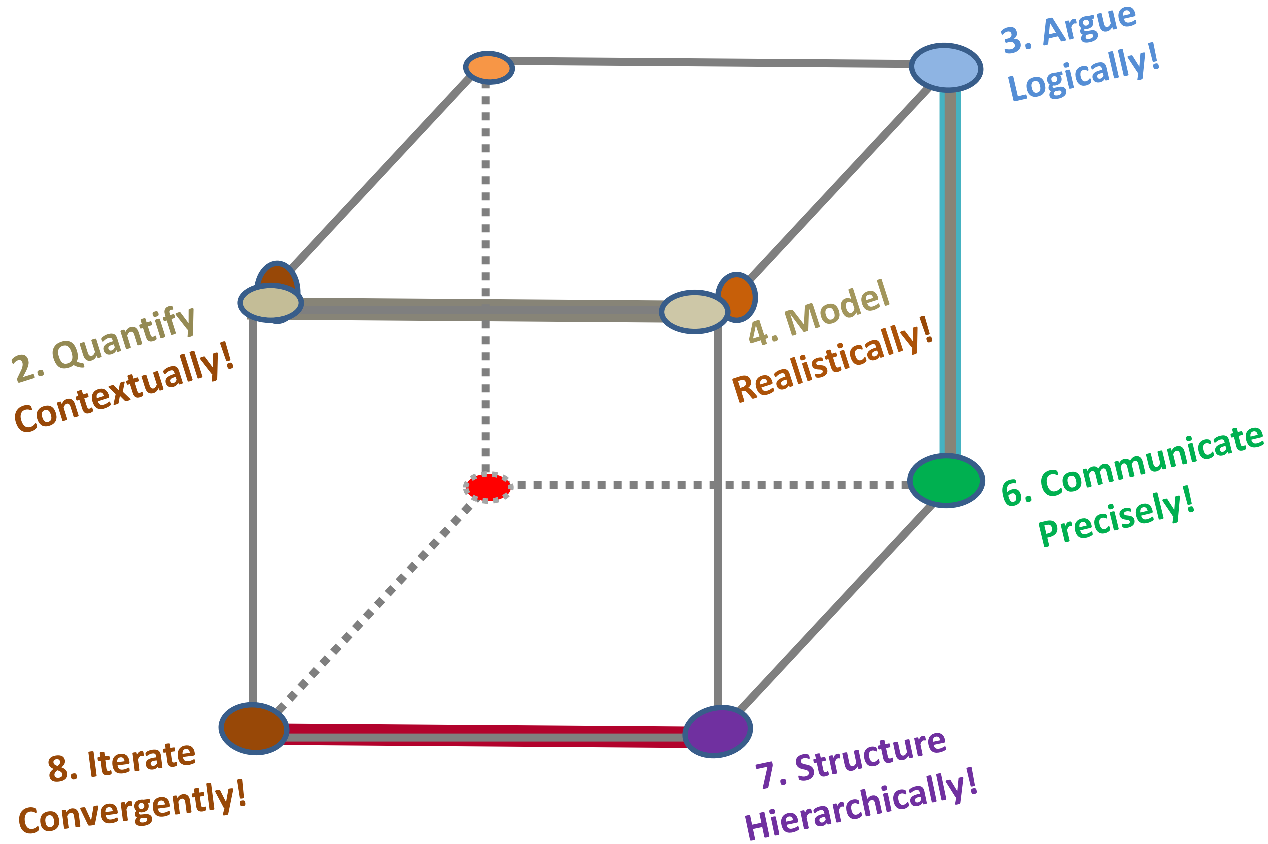
1. Solve Tenaciously! **Orange** is ardent, and intense.
2. Quantify Contextually! **Gray** is abstract whereas **Brown** is concrete.
3. Argue Logically! **Blue** is clear-eyed and deep.
4. Model Realistically! **Silver** is theoretical whereas **Gold** is practical.
5. Tool Incisively! **Red** is for halting where appropriate.
6. Communicate Precisely! **Green** is for carrying explanations forward.
7. Structure Hierarchically! **Purple** is for exacting limitations.
8. Iterate Recursively! **Brown** is for dense continuities.

Three Affine Edges:

Quantifying + Modeling = **Engineering & Science.**

Iteration + Structure = **Theoretical Mathematics.**

Rational Argument + Precise Expression = **"Expert Testimony."**



1. *Solve Tenaciously!*

Analyze your problem's givens, constraints, relationships, goals, special cases, simpler versions. Monitor progress and check your answers by another method.

Persistence produces timely inventions and insights.

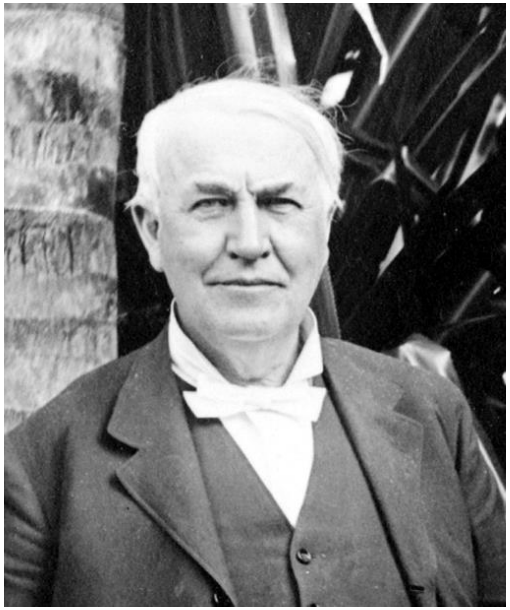
- Mathematics is stick-to-it-ive-ness.
- Mathematics is the etymological definition of study—Greek μάθημα(mathema), meaning "subject of instruction.“
- Mathematics is the paradigm discipline of the liberal arts—no royal road to mathematics, just hard study.
- Students are studious, zealots are zealous, disciples are mathoi, disciples are disciplined.

1. *Grown-ups Solve Tenaciously*

"Genius is one per cent inspiration,
ninety-nine per cent perspiration."

"Those weren't failures. We discovered
2000 ways you can't make a light bulb."

"Hell, there are no rules here –
we're trying to *accomplish* something!"



Thomas Edison
1847-1951

Practical uses for electricity: stock ticker, gramophone, movies, light bulbs

1. Rich Spiral Lessons Engage

Tenacious Problem Solving

- Problem sets with dramatically short and long problems.
- Problem sets with dramatically hard and easy problems.
- Problem sets that mix similar-looking problems, some of which actually cannot be solved by the methods you explain in class!

How is this possible?
The answer is HOMEWORK! Flipped Homework!

Picture a monster 20-page homework packet given out at the beginning of a 3 or 4 week topic. You lecture selected problems in class; the whole packet's due, one-big-final-perfect-revision, same day as the monthly exam.

2. *Quantify Realistically!*

Manipulate symbols as if they have a life of their own, but pause to probe referents for the symbols. Consider the units involved.

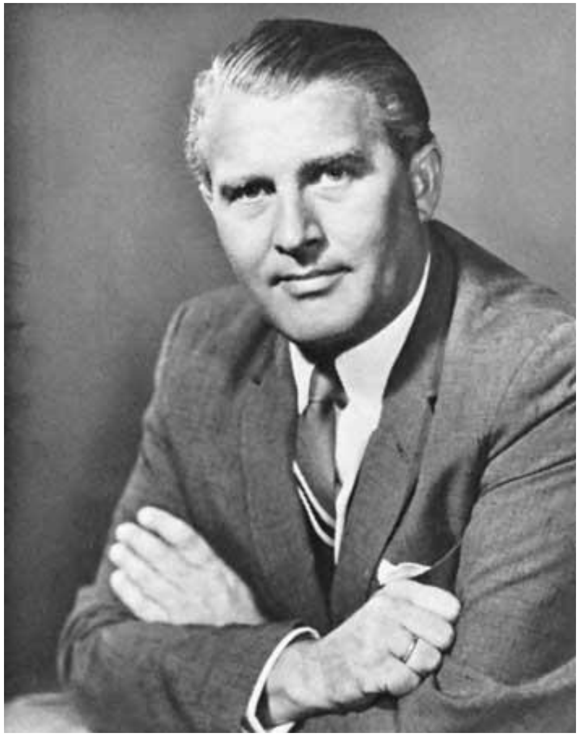
> **Quantifying** is the 'word-problem' pattern:
> let x equal; solve-for-x; restate-physical-units, etc.

- *Decontextualizing* asserts: "Doing manual algorithms fluently is fun."
- *Contextualizing* asserts: "Physical /social realities always contain provocative numeric relationships."
- Ratio and proportion, especially dimensional analysis of units of measure, are at the core of quantifying realistically.
- Quantifying/Abstracting is the 'little brother' of Modeling.

2. Grown-Ups Quantify Realistically

"Hermann Oberth was the first, who when thinking about the possibility of spaceships grabbed a slide-rule and presented mathematically analyzed concepts and designs."

--von Braun in praise of his college professor of Aeronautical Engineering



Werner von Braun
1912-1977

Practical rocketry from V-2 to Apollo

2. Rich Spiral Lessons Engage

Realistic Quantification

- Miles per gallon for cars, thousands of pounds per hour for airliners, tons per day for cargo ships.
- Does a family of 4 use more fuel driving to California or flying?
- How many horsepower in an aircraft carrier?
- How many horsepower in a toaster?
- What is a foot-pound anyway? Are there foot-pounds in a toaster?
- How come they charge you for kilowatt-hours, not for kilowatts?
- What does a mole of caffeine weigh, and how many cups of coffee is that?
- Bring back the slide rule?

3. *Argue Logically!*

Use established results and logical statements to explore conjectures.
Communicate your conclusions and justify them to others.

Reason justifies irrefutable conclusions.

- School logic is primarily deductive—syllogisms, premises, conclusions.
- The logic practice mentions inductive reasoning, but content standards for inductive reasoning are descriptive—confidence levels not involved.
- Logic is not an exalted practice; logic is subordinated to making convincing arguments and analyzing arguments of others.
- Logic is an LSAT kind of thing.
- Logical Argumentation is the ‘conservative twin’ of Precise Communication.

3. Grown-Ups Argue Logically

"You can draw a straight line from any point to any point." --Postulate 1, Book 1

Euclid of Alexandria
325-265 BC

Axiomatic geometry and number theory

3. Rich Spiral Lessons Engage

Logical Argumentation

- *"There are numbers that are not rational." 8NS*
- Reductio ad absurdum is not a dirty word.

- *"Prove" that* solving systems of equations by elimination works.
- The dual of a problem is always an interesting problem.

- The Annual High School Mock Trial Contests, Ohio Center for Law-Related Education, engage reasoning from laws, facts and principles—why should Social Studies teachers have all the fun?

4. *Model Realistically!*

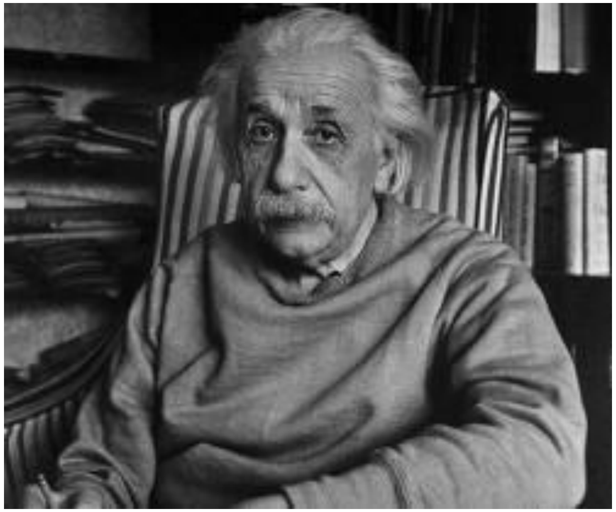
Apply mathematics to everyday life and society. Use functions, diagrams and formulas to model real quantities and draw practical conclusions.

Modeling is the social-law/natural-law pattern:
bell-shaped distributions, laws of motion, etc.

- Mathematical laws of life, society and nature are the holy grail of scientific inquiry.
- But in the social sciences, modelers are the dismal scientists: pollsters, actuaries, statisticians, economists, efficiency experts.
- Nevertheless, learning how classical functions apply to real social systems is <u>the</u> core mathematical preparation for careers and college.
- Modeling Realistically is the ‘big brother’ of Quantifying Contextually.

4. Grown-Ups Model Realistically

"As far as the laws of mathematics refer to reality, they are not certain, as far as they are certain, they do not refer to reality.

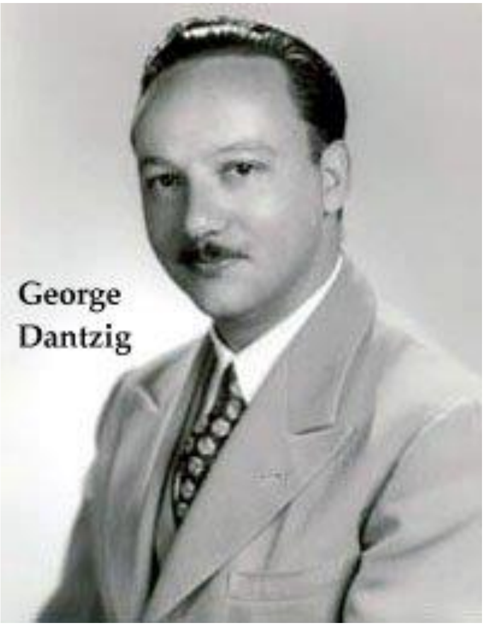


Albert Einstein
1879-1955

Special relativity, general relativity and basic quantum theory

4. Grown-Ups Also Model Realistically

"The final test of a theory is its capacity to solve the problems which originated it."



George Dantzig
1915-2005

Linear programming and the simplex algorithm

4. Rich Spiral Lessons Engage

Realistic Modeling

- Scaling social quantities from the personal level to the community level is a fundamental aspect of middle-school math.
- In high school, bell-shaped classroom data distributions suited to both manual and spreadsheet graphing are everywhere to be found.
- In Algebra 1 and Algebra 2, the Simplex Method is eminently lesson-able, middle of the first semester.

5. *Tool Incisively!*

Consider pencil and paper, concrete models, computers, spreadsheets and calculators. Use external websites to pose and explore problems.

Tools magnify and focus human effort.

- Tool use is always tactical, not strategic. I call tool use incisive.
- Using hand-held calculators appropriately requires a moral and ethical decision to always value numerical thought over numerical labor.
- Computers are not just another math tool. Computers are the core tool and glue of our global civilization. And likely to remain so.
- How computers look and feel is different from how computers work. The only way to learn how computers work is to learn a computer language. Another language, another self.

5. *Grown-Ups Tool Incisively*

"A man provided with paper, pencil, and eraser, and subject to strict discipline, is in effect a universal machine."

Alan Turing
1912-1954

Theory and practice of finite-state computing machines

5. Rich Spiral Lessons Engage

Incisive Tool Use

- Knowing when and how to use the right kind of graph paper is deeper than most people think.
- Euclidean compass and straightedge constructions are deeper than most people think.
- WHEREAS: Most people in our school or neighborhood—students, parents, teachers, administrators—have never written a computer program in a universal programming language;
- AND WHEREAS: Visual Basic for MS-Office is a universal programming language and debug environment already installed on our PCs and Macs, home and school;
- RESOLVED: Lessons will be involved! One million prime numbers in four seconds, anyone? We can teach this!
- WANTED: One-semester core elective computer programming for almost everybody. AP Java here we come!

6. Communicate Precisely!

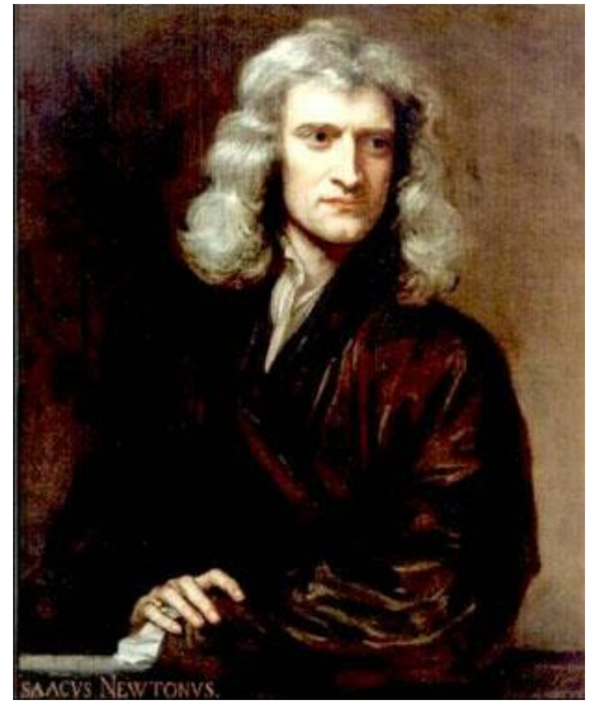
Formulate explanations and examine claims using stated symbols and explicit definitions. Specify units of measure; use appropriate numerical precision.

Precise Expression guarantees compelling communication.

- In reality, mathematical objects are fuzzier than they appear; most math writing is even fuzzier.
- Math researchers write jargon for insiders; textbook authors decorate pages with goofy graphics.
- Effective explanations require equal attention to the most precise details and the widest possible audience.
- Precise Communication is the ‘liberal twin’ of Logical Argumentation.

6. Grown-Ups Communicate Precisely

"I do not feign hypotheses! Gravity really exists, acts according to the laws that we have set forth, and is sufficient to explain all the motions of the heavenly bodies and of our seas." –General Scholium, Book 3



Isaac Newton
1642-1727

Inertia, gravity, calculus

6. Rich Spiral Lessons Engage

Precise Communication

- Kids speaking and writing math for their K-12 peers is the end-all/be-all classroom activity for careers and college.
- You're a kid who can solve a system of equations two ways. Now can you write a system of equations other kids will want to solve in a particular way? Can you deal with it when kids tell you your system is corny, or plain wrong?
- How about that blockbuster 20-day homework packet you've been working on? Can you make up a problem set like that?
- Can you devise a sample chapter exam including solution and grading rubric?
- AT LEAST can't we always trade, grade and discuss the bellwork together every day?

7. Structure Hierarchically!

Discern significant lines in a geometric figure and draw auxiliary lines.
View an algebraic expression as one object or a composition of significant objects.

Structuring is the finite-math pattern:
graph theory, prime number theory, and so forth.

- Seeing significant structures in complex situations is a deep and wacky skill—like catching lightening in a bottle.
- Darwinian evolution is the only great scientific theory based on qualitative rather than quantitative insights.
- Structuring is the finite analytical brother of Iterating.

7. *Grown-Ups Structure Hierarchically*

"The further a mathematical theory is developed, the more harmoniously and uniformly does its construction proceed, and unsuspected relations are disclosed between hitherto separated branches of the science."

David Hilbert
1862-1943

Proof theory, mathematical logic, axiomatic geometry

7. Rich Spiral Lessons Engage

Hierarchical Structure

- If you live in Koenigsberg, Is it possible to return home by crossing the seven bridges of Koenigsberg exactly once each?
- A cube has 8 vertices, 6 faces and 12 edges, so ***V + F – E = 2.*** Is this relationship true for a very large class of faceted solids?
- Prove there are exactly five regular solids.
- How many different ways can you arrange 8 distinct colors at the corners of a Praxis Cube anyway?
- Why is paper folding so much fun anyway?

8. Iterate Recursively!

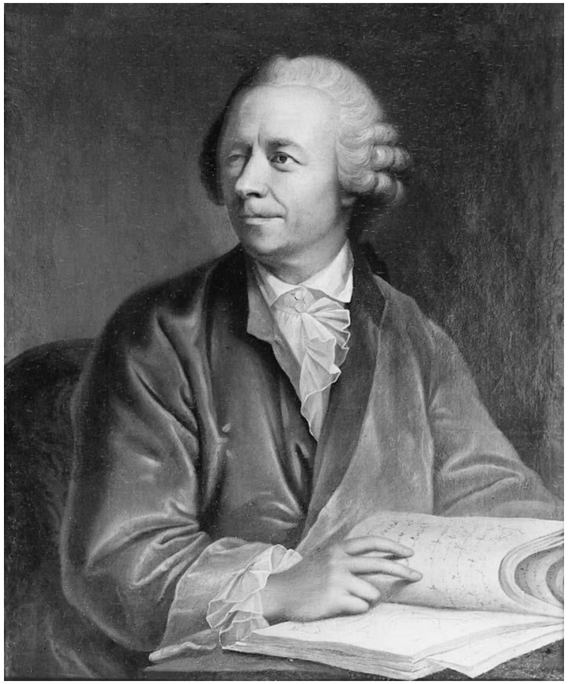
When your calculations repeat, seek general methods and cumulative shortcuts. Investigate reasonableness of your problem-solving process while attending to its details.

Iterating is the continuous-math pattern:
convergent infinite series, calculus and so forth.

- The natural numbers and the real continuum are different orders of infinity. Points on a line and points on a plane are the same order of infinity.
- Some infinite calculations yield finite results, and vice versa.
- Commutative and associate laws do not always apply when adding the terms of an infinite series.
- Iterating is the continuous analytical brother of Structuring.

8. Grown-Ups Iterate Recursively

"Nothing takes place in the world whose meaning is not that of some maximum or minimum."



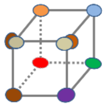
Leonard Euler
1707-1783

Complex infinitesimal calculus, graph theory

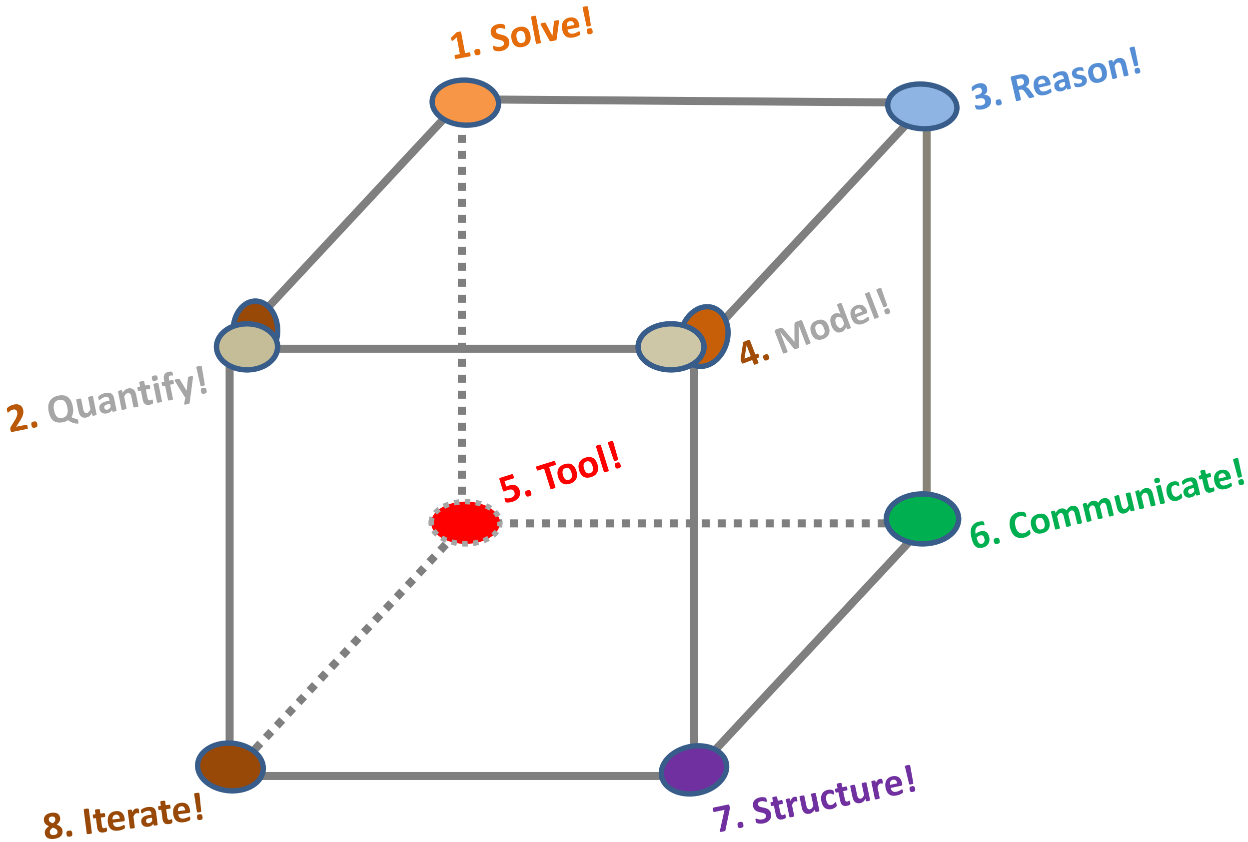
8. Rich Spiral Lessons Engage

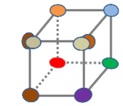
Recursive Iteration

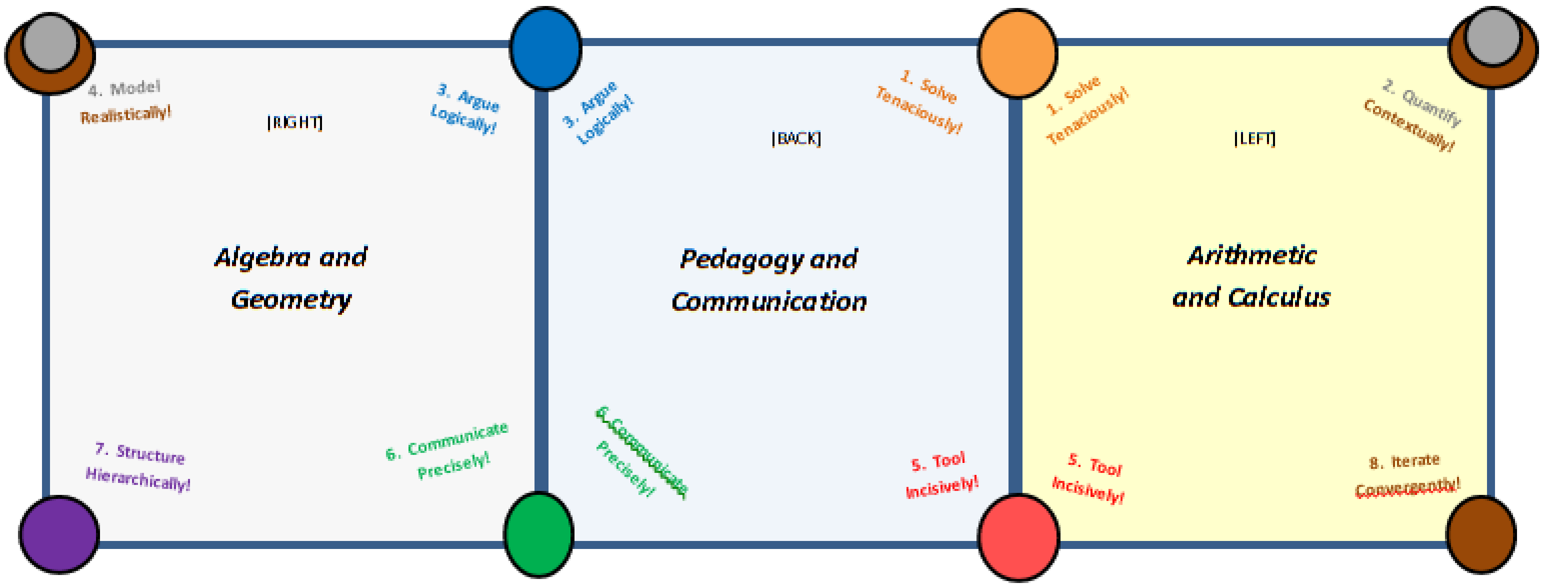
- Repeating decimals are the first infinite power series in middle school. However 1/3 does not repeat in base$_3$!
- Simple geometric series converge; simple harmonic series don't; 9th graders can see the difference through repeated reasoning.
- Archimedes' Method of Exhaustion can approximates $\boldsymbol{\pi}$ visually with Geogebra, numerically with Excel formulas.
- Numerical differentiation on a family of exponential functions can approximate Euler's ***e*** using Geogebra, Excel and Visual Basic.



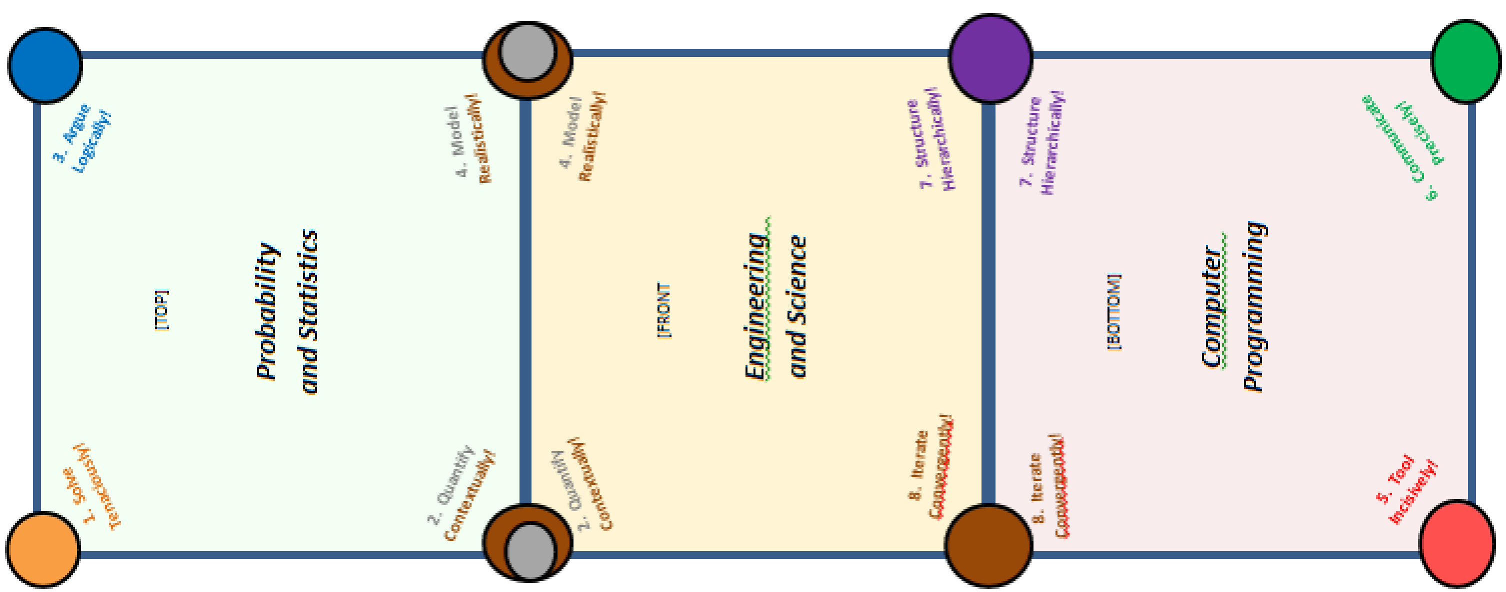
Ohio 2017 Standards for Mathematical Practice







Ohio 2017 Standards for Mathematical Practice—Cubed!



Ohio 2017 Standards for Mathematical Practice

Solve! Analyze your problem’s givens, constraints, relationships, goals, special cases simpler versions. Monitor progress and check your answers by another method.

Quantify! Manipulate symbols as if they have a life of their own, but pause to probe referents for the symbols. Consider the units involved.

Argue! Use established results and logical statements to explore conjectures. Communicate your conclusions and justify them to others.

Model! Apply mathematics to everyday life and society. Use functions, diagrams and formulas to model real quantities and draw practical conclusions.

Tool! Consider pencil and paper, concrete models, computers, spreadsheets and calculators. Use external websites to pose and explore problems.

Communicate! Formulate explanations and examine claims using stated symbols and explicit definitions. Specify units of measure and use appropriate numerical precision.

Structure! Discern significant lines in a geometric figure and draw auxiliary lines. View an algebraic expression as one object or a composition of significant objects.

Iterate! When your calculations repeat, seek general methods and cumulative shortcuts. Investigate reasonableness of your problem-solving process while attending to its details.

Ohio 2017 Standards for Mathematical Practice

Solve! Analyze your problem's givens, constraints, relationships, goals, special cases, simpler versions. Monitor progress and check your answers by another method.

Quantify! Manipulate symbols as if they have a life of their own, but pause to probe referents for the symbols. Consider the units involved.

Argue! Use established results and logical statements to explore conjectures. Communicate your conclusions and justify them to others.

Model! Apply mathematics to everyday life and society. Use functions, diagrams and formulas to model real quantities and draw practical conclusions.

Tool! Consider pencil and paper, concrete models, computers, spreadsheets and calculators. Use external websites to pose and explore problems.

Communicate! Formulate explanations and examine claims using stated symbols and explicit definitions. Specify units of measure and use appropriate numerical precision.

Structure! Discern significant lines in a geometric figure and draw auxiliary lines. View an algebraic expression as one object or a composition of significant objects.

Iterate! When your calculations repeat, seek general methods and cumulative shortcuts. Investigate reasonableness of your problem-solving process while attending to its details.

THANKYOU!