

BGSU Mathematics Competition
March 24 2018 **A** (Above Calculus III)

No cell phones are allowed. Show all your work. Justify your answers.

1) Let x and y be two numbers such that $x + y = xy = 17$. Evaluate $(x^2 - 17x)(y + \frac{17}{y})$.

2) Find the units digit of $3^{3018} - 2^{2018}$.

3) Find the sum:

$$S = \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \frac{1}{4 \cdot 6} + \dots + \frac{1}{2014 \cdot 2016} + \frac{1}{2015 \cdot 2017} + \frac{1}{2016 \cdot 2018}$$

4) The local weatherman was recently asked if he could recall the seven day forecast from memory. He replied that he could not remember the exact forecast, but that he remembered that each day had a different temperature and that the product of the temperatures was 180. What are the seven temperatures in the forecast? (Assume that the individual daily temperatures are rounded to the ones place: for example 23.56 degrees would be rounded to 24 degrees).

5) Prove that the Fibonacci sequence satisfies the identity:

$$F_{3n} = F_{n+1}^3 + F_n^3 - F_{n-1}^3$$

for all $n \geq 1$. (The Fibonacci sequence F_n is defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$ for $n \geq 1$.)

6) Prove that any positive integer can be represented as $\pm 1^2 \pm 2^2 \pm 3^2 \pm \dots \pm n^2$ for some positive integer n and some choice of the signs.

7) $\log_{20}(18) = a$ and $\log_{20}(12) = b$. what is the value of $\log_{20}(27)$ in terms of a and b ?

8) Prove that there is an integer of the form 111...111 divisible by 2017.

Registration 2018 BGSU Mathematics Competition;

Your NAME:

E-mail:

(Optional)

Math class you are taking this semester/year:

Name of your instructor(s):

1)

2)

3)

4)

5)

6)

7)

8)

Total: