BGSU Mathematics Competition
March 25 2017 **A** (Above Calculus III)

No cell phones are allowed. Show all work. Explain your answers.

1) Find the sum:

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{2015\cdot 2016} + \frac{1}{2016\cdot 2017}$$

(Hint: Find A and B such that $\frac{1}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1}$)

2) Find the largest value of n for which $8^{20}15^{17}17^{20}$ is divisible by 10^n.

3) Let α, β and γ, be the angles of a triangle. If $sin(\alpha)$, $sin(\beta)$, and $sin(\gamma)$ are all rational, prove that $cos(\alpha)$, $cos(\beta)$, and $cos(\gamma)$ are also rational.

4) Find the sum

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n-2}{(n-1)!} + \frac{n-1}{n!}$$

5) A 2×3 rectangle has vertices as $(0,0)$, $(2,0)$, $(0,3)$, and $(2,3)$. It rotates $90°$ clockwise about the point $(2,0)$. It then rotates $90°$ clockwise about the point $(5,0)$, then $90°$ clockwise about the point $(7,0)$, and finally, $90°$ clockwise about the point $(10,0)$. (The side originally on the x-axis is now back on the x-axis.) Find the area of the region above the x-axis and below the curve traced out by the point whose initial position is (1,1).

6) Let R be the region consisting of the points (x,y) of the cartesian plane satisfying both $|x| - |y| \leq 1$ and $|y| \leq 1$. Sketch the region R and find its area.

7) Show that there is no matrix with real entries $A \in M_2(\mathbb{R})$ such that

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

8) Show how to cut a 9×16 rectangle into two pieces that can be assembled into a 12×12 square.

Registration 2017 BGSU Mathematics Competition;

Your NAME:

E-mail:

(Optional)

Math class you are registered:

Name of your instructor:

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1)

2)

3)

4)

5)

6)

7)

Total: