

BGSU Mathematics Competition
April 2 2016 **A**

No cell phones are allowed. Show all work. Explain your answers.

1) Solve:

$$\left(\left(\left(\left(81 + x\right)^{\frac{1}{3}} + x\right)^{\frac{1}{3}} + x\right)^{\frac{1}{3}} + x\right)^{\frac{1}{3}} = 1$$

2) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(3n)!}{n!(2n)!} x^{2n}$.

3) Find the area of the largest equilateral triangle that fits in a square of side length 1.

4) Given that $f(0) = 0$, $f'(0) = 1$, $f''(0) = 3$, and $f'''(0) = 5$, find $\int_0^1 x f'''(2x) dx$.

5) You and two other friends have purchased a 45 ounce pitcher of soda (45 ounces fills the pitcher to the brim). Unfortunately you were sent with three different sized cups. One holds 25 ounces, one holds 20 ounces and one holds 10 ounces. How could you distribute the soda evenly without spilling a drop?

6) Find the sum:

$$2 + 4 + 6 + 8 + \dots + 2014 + 2016$$

7) Show that for a positive integer n , we have that $(n + 2)(n + 1)n(n - 1) + 1$ is the square of an integer.

8) Five boxes of different but unknown weights arrived in the USPS office. Mary was assigned the job of determining their respective weights. Unfortunately, all of the boxes weigh less than 100 pounds, and the scale available to her reads only weights over 100 pounds. Mary decides to weigh the boxes in pairs so that each box is weighted with every other box. The weights of all possible pairs are 110, 112, 113, 114, 115, 116, 117, 118, 120, and 121 pounds. What are the weights of the five boxes?

Registration 2016 BGSU Mathematics Competition;

Your NAME:

(Optional) Math class you are registered, and name of your instructor: