BGSU Mathematics Competition
March 21 2015 B

No cell phones are allowed. Show all work. Explain your answers.

1) Consider the list of ten statements below, each claiming the next one is false. How many of the ten statements is/are true?

1) Statement 2 is false.
2) Statement 3 is false.
3) Statement 4 is false.
...
9) Statement 10 is false.
10) Statement 1 is false.

2) Use the picture on the back page to decipher the following message:

EBOX EHGZ TGW IKHLIXK

3) If $2^x = 15$ and $15^y = \frac{1}{32}$, determine the value of xy.

4) Suppose that at noon a train leaves Chicago for New York traveling at 60 miles per hour, and at the same time a train leaves New York for Chicago also traveling at 60 miles per hour on the same train track. Also at noon, a fly starts flying back and forth between the two trains at 90 miles per hour, starting from the front of the train departing Chicago, and the fly continues to fly back and forth between the two trains until it is squashed when the two trains collide. Assume that the train track from Chicago to New York is 800 miles long and lies along a straight line and the fly also travels along a straight line. Determine the total distance traveled by the fly.

5) Prove that for any integer n the number $\frac{n^5-5n^3+4n}{120}$ is an integer.

6) Is it possible to write 1 as a sum $\frac{1}{n_1} + \frac{1}{n_2} + ... + \frac{1}{n_6} = 1$ where n_i are positive odd integers?

7) Let $N = 9 + 99 + 999 + ... + \overbrace{99...9}^{99}$. Determine the sum of digits of N.

8) Let $f(x) = e^{x^2} sin(x)$. Find with proof $f^{(2014)}(0)$ (the 2014-th derivative of f at 0).