BGSU Mathematics Competition March 22 2014 **A**

No cell phones are allowed. Show all work. Explain your answers.

1) Find the polynomial P(x) for which there exists a natural number n such that $P(P(x)) = x^n - 1$

2) Let $f(X) = X^2 + 12X + 30$. Find x such that f(f(f(f(x)))) = 0.

3) Let f and g be real-valued functions of a real variable defined by $f(x) = Ae^{Bx} + C$ and $g(x) = ae^{bx} + c$ where A, B, C, a, b, c are real numbers. Determine (with proof) the maximum possible number of points of intersection of the graphs of f and g.

4) If all 720 permutations of 1, 2, 3, 4, 5 and 6 are arranged in numerical order what is the 417-th term?

5) You have two pices of fuse, each of which burns in 1 minute. Use these pices of fuse to time 45 seconds. You may not use scissors, and the rate of burning may vary along the fuse.

6) Two lines are tangent to a circle of radius ON = r and intersect at a point a distance OM = d from the center of the circle as shown below. Determine the area of the shaded region enclosed by the lines and the circle.



Registration 2014 BGSU Mathematics Competition;

Your NAME:

(Optional) Math class you are registered, and name of you instructor: