BGSU Mathematics Competition
March 16 2013

B

No cell phones, calculators, etc. are allowed.

1) What is the only real value of x which satisfies $20^x \cdot 13^x = 2013^x$.

2) A jar contains only quarters and coins of less values. The average value of these coins is 16 cents. Adding a quarter to the jar raises the average value in the jar to 17 cents. Before this quarter was added, how many quarters were in the jar?

3) If x and y are non-zero real numbers such that $|x|+y = 3$ and $|x|y+x^3 = 0$ then determine the value of $x - y$.

4) Find the value of the sum

$$1 + 4 + 7 + 10 + ... + 2014$$

5) It is known that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. Find the value of the sum $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

6) Consider a triangle ABC such that $|AB| = 3$, $|AC| = 4$ and $|BC| = 5$. Find the radius of the inscribed circle.

7) Farmer Bob has 5^{2013} eggs. He packs them into cartons, each of which holds a dozen eggs, until he no longer has enough eggs to fill a carton. Then he takes the leftover eggs and makes an omelette. How many eggs go into his omelette?