

# BGSU Mathematics Competition

## March 16 2013

**B**

**No cell phones, calculators, etc. are allowed.**

1) If  $x + x^{-1}$  is an integer, prove that  $x^{2013} + x^{-2013}$  is also an integer.

2) A jar contains only quarters and coins of less values. The average value of these coins is 16 cents. Adding a quarter to the jar raises the average value in the jar to 17 cents. Before this quarter was added, how many quarters were in the jar?

3) If  $x$  and  $y$  are non-zero real numbers such that  $|x| + y = 3$  and  $|x|y + x^3 = 0$  then determine the value of  $x - y$ .

4) Pick at random a 4-digit PIN. What is the probability that the chosen PIN has two equal consecutive digits (such as, for example, 0558).

5) It is known that  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ . Find the value of the sum  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ .

6) Consider a triangle  $ABC$  such that  $|AB| = 3$ ,  $|AC| = 4$  and  $|BC| = 5$ . Find the radius of the inscribed circle.

7) Farmer Bob has  $5^{2013}$  eggs. He packs them into cartons, each of which holds a dozen eggs, until he no longer has enough eggs to fill a carton. Then he takes the leftover eggs and makes an omelette. How many eggs go into his omelette?

8) Find the sum

$$\frac{1}{2 \cdot 5} + \frac{1}{3 \cdot 6} + \frac{1}{4 \cdot 7} + \frac{1}{5 \cdot 8} + \dots$$

9) Find  $2 \times 2$  matrices  $A$  and  $B$  such that

$$AB - BA = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

or show that such matrices do not exist.