

BGSU Mathematics Competition  
March 25th 2023 **A** (above Calculus II)

**No cell phones are allowed. Show all your work. Justify your answers.**

- (1) Solve the equation

$$4^x + 6^x = 9^x.$$

- (2) Decide which of the following numbers is larger  $215^{223}$  or  $223^{215}$ .

- (3) A commuter is in the habit of arriving at his suburban station each evening exactly at 5 PM. His wife always meets the train and drives him home. One day he takes an earlier train, arriving at the station at 4 PM. The weather is pleasant, so instead of telephoning home he starts walking along the route always taken by his wife. They meet somewhere on the way. He gets into the car and they drive home, arriving at their house ten minutes earlier than usual. Assuming that the wife always drives at a constant speed, and that on this occasion she left just in time to meet the 5 PM train, can you determine how long the husband walked before he was picked up?

- (4) Find the sum:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{2021 \cdot 2022} + \frac{1}{2022 \cdot 2023}$$

(Hint: Find  $A$  and  $B$  such that  $\frac{1}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1}$ )

- (5) Find the primes  $a, b, c$  such that

$$51a + 24b + 17c = 2023.$$

- (6) Let  $f(X) = X^2 + 12X + 30$ . Find  $x$  such that  $f(f(f(f(f(x)))))) = 0$ .

- (7) Let  $R$  be the region consisting of the points  $(x, y)$  of the cartesian plane satisfying both  $|x| - |y| \leq 1$  and  $|y| \leq 1$ . Sketch the region  $R$  and find its area.

- (8) Show how to cut a  $9 \times 16$  rectangle into two pieces that can be assembled into a  $12 \times 12$  square.

**Registration 2023 BGSU Mathematics Competition;**

**Your NAME:**

E-mail:

(Optional)

Math class you are taking this semester/year:

Name of your instructor(s):

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1)

2)

3)

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7)

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Total: