Welcome to …

(CO)^{2}MP Elementary

Common Core for Mathematical Proficiency in Elementary Schools

Agenda

- Number Talk
- Algebra – Addition and Subtraction
- About the Standard Algorithms
- Lunch
- Connecting Arithmetic to Algebra
- What does it mean to Subtract?
- Subtraction Methods
- Reflection

Morning Warm-Up

63 + 28

Early Algebra Operations – Addition and Subtraction

Well planned explorations about operations provide spaces for relational thinking and algebraic reasoning.

The CCSSM discusses the following problem types:

- Adding to
- Taking from
- Putting Together/Taking Apart
- Comparing

Early Algebra Operations – Addition and Subtraction

Read the following three problems and explain how they are alike and different.

A. Three marbles were in a cup. Someone put two more marbles in the cup. How many marbles are in the cup now?

B. Roger put seven marbles in a cup. Marcy put some more marbles in the cup. Then there were fifteen marbles. How many marbles did Marcy put in?

C. Some marbles were in a cup. Shawn put 38 more marbles in the cup. Then there were 74 marbles. How many marbles were in the cup before Shawn put his marbles in the cup?

Early Algebra Operations – Addition and Subtraction

Result Unknown	**A. Three marbles were in a cup. Someone put two more marbles in the cup. How many marbles are in the cup now?** **3 + 2 = ? (Kindergarten)**
Change Unknown	**B. Roger put seven marbles in a cup. Marcy put some more marbles in the cup. Then there were fifteen marbles. How many marbles did Marcy put in?** **7 + ? = 15 (First)**
Start Unknown	**C. Some marbles were in a cup. Shawn put 38 more marbles in the cup. Then there were 74 marbles. How many marbles were in the cup before Shawn put his marbles in the cup?** **? + 38 = 74 (Second)**

Early Algebra Operations – Addition and Subtraction

Read the following three problems and explain how they are alike and different.

X. Fourteen cupcakes were on the tray. Kendra ate some of them. Then there were nine. How many cupcakes did Kendra eat?

Y. Some small cupcakes were on a tray. Blanca ate eleven of them. Then there were forty three small cupcakes. How many small cupcakes were on the tray before?

Z. Four cupcakes were on the tray. Bart ate three of them. How many cupcakes are on the tray now?

Early Algebra Operations – Addition and Subtraction

Result Unknown	**Z. Four cupcakes were on the tray. Bart ate three of them. How many cupcakes are on the tray now?** **4 – 3 = ? (Kindergarten)**
Change Unknown	**X. Fourteen cupcakes were on the tray. Kendra ate some of them. Then there were nine. How many cupcakes did Kendra eat?** **14 - ? = 9 (First)**
Start Unknown	**Y. Some small cupcakes were on a tray. Blanca ate eleven of them. Then there were forty three small cupcakes. How many small cupcakes were on the tray before?** **? – 11 = 43 (Second)**

Early Algebra Operations – Addition and Subtraction

Read the following four problems and explain how they are different from A, B, C and X, Y, Z:

Q. Four brown puppies and one black puppy are in the dog house. How many puppies are in the dog house?

R. Twelve puppies are in the kennel. Three are black and the rest brown. How many puppies are brown?

S. Twelve puppies are in the kennel. Some are black and seven are brown. How many puppies are black?

T. The kennel has seven puppies. How many can be put in the green pen and how many can be put in the orange pen?

Early Algebra Operations – Addition and Subtraction

Result Unknown	**Q. Four brown puppies and one black puppy are in the dog house. How many puppies are in the dog house?** **4 + 1 = ? (Kindergarten)**
Addend Unknown	**R. Twelve puppies are in the kennel. Three are black and the rest brown. How many puppies are brown?** **3 + ? = 12 (First)**
Other Addend Unknown	**S. Twelve puppies are in the kennel. Some are black and seven are brown. How many puppies are black?** **? + 7 = 12 (First)**
Both Addends Unknown	**T. The kennel has seven puppies. How many can be put in the green pen and how many can be put in the orange pen?** **? + ? = 7 (First - explore possibilities less than ten)** **0 + 7 = 7, 1 + 6 = 7, 2 + 5 = 7, 3 + 4 = 7**

Early Algebra Operations – Addition and Subtraction

Read the following six problems and explain how they are different the previous problems.

D. John has 8 oranges and Mark has 13. How many more oranges does Mark have than John?

E. John has 8 oranges and Mark has 13. How many fewer oranges does John have than Mark?

F. Mark has 15 more oranges than John. John has 4 oranges. How many oranges does Mark have?

G. John has 15 fewer oranges than Mark. John has 4 oranges. How many oranges does Mark have?

H. John has 14 more oranges than Mark. John has 26 oranges. How many oranges does Mark have?

I. Mark has 14 fewer oranges than John. John has 26 oranges. How many oranges does Mark have?

Early Algebra Operations – Addition and Subtraction

Difference Unknown	**D. John has 8 oranges and Mark has 13. How many more oranges does Mark have than John?**	**E. John has 8 oranges and Mark has 13. How many fewer oranges does John have than Mark?**
	8 + ? = 13 or 13 – 8 = ? (First)	
Bigger Unknown	**F. Mark has 15 more oranges than John. John has 4 oranges. How many oranges does Mark have?**	**G. John has 15 fewer oranges than Mark. John has 4 oranges. How many oranges does Mark have?**
	4 + 15 = ? Or 15 + 4 = ? (First)	
Smaller Unknown	**H. John has 14 more oranges than Mark. John has 26 oranges. How many oranges does Mark have?**	**I. Mark has 14 fewer oranges than John. John has 26 oranges. How many oranges does Mark have?**
	26 – 14 = ? Or ? + 14 = 26 (Second)	

Early Algebra Operations – Addition and Subtraction

Four Problem Types for Addition and Subtraction

1. Add to
2. Take from
3. Put together/Take apart
4. Compare

So what are the *relationships* that each of the four problem types encourage thinking about?

$3 + 4 = n$ $\quad$ $n + 4 = 7$ $\quad$ $3 + n = 7$

$4 + 3 = n$ $\quad$ $7 - n = 3$ $\quad$ $7 - n = 4$

$7 - 4 = n$ $\quad$ $7 - 3 = n$

For any whole numbers, a, b, c

$a + b = c$ also means $b = c - a$ or $a = c - b$

Early Algebra Operations – Addition and Subtraction

Throughout the structure of the CCSSM we see a clear delineation of both the importance and the order of student derived and represented ways of doing mathematics and the “standard algorithm.”

This is made evident both in ***when*** the expected proficiency with the standard algorithm is declared and by ***how*** the language of the standards reads.

Early Algebra Operations – Addition and Subtraction

Look through the CCSSM and find an example of ***when*** the expected proficiencies with the standard algorithms for addition, subtraction, multiplication, and division are declared. Write down the standards identifying information.

Look through the CCSSM and find an example of ***how*** its language indicates the importance of student derived and represented ways of doing mathematics come prior to learning about the standard algorithm. Write the entire standard and identify the significant words by underlining them.

Early Algebra Operations – Addition and Subtraction

An example of ***when*** can be easily seen in the fact that we start teaching addition and subtraction in Kindergarten. As the grade levels increase the numbers are enlarged and we expect explanation of different ways to add and subtract. It is not until 4th grade (4.NBT.4) that we see the first mention of the expectation for fluency with the standard algorithms.

Examples of ***how*** the language demonstrates this can be found in 1.OA.1 "Use addition and subtraction within 20 to solve word problems…by using objects, drawings, and equations with a symbol for the unknown number to represent the problem" and 2.NBT.9 "Explain why addition and subtractions work, using place value and the properties of operations where explanations may be supported by drawings or objects."

Lunch

Connecting Arithmetic to Algebra

"Articulating general claims is essential to investigating generalizations. As students put their ideas into words, they clarify their ideas, develop common language, and come to a common understanding of exactly what their general claim is. Once they have agreed what it is they are trying to prove, they can work together to justify their claim."

- Russell, Shifter, Bastable (2011)

Connecting Arithmetic to Algebra

Read page 38 to page 46.

As you are reading, highlight, underline, or make note of things you find significant or meaningful.

When your group is finished, take some time to discuss these things you highlighted, underlined, or made a note of.

Connecting Arithmetic to Algebra

Focus Questions #1 and #3 (pg. 51)

1. Consider Mr. Williams' grade 1 classroom narrative in which he works with his class on noticing and articulating the commutative property of addition.

- What do you notice about the questions that Mr. Williams asks?
- What is the impact of his questions on his students?
- What ideas about posing questions does this bring up for you as you consider your own teaching practice?

3. In the grade 3 classroom narrative, Ms. Kaye's students identify number patterns as they look at sequences of arithmetic expressions. However, finding number patterns is not the end of the process.

- What happens in this case, after the patterns have been noted?
- What math ideas are the focus of the work and how are those ideas related to noticing patterns?
- What questions does this raise about the math work you want your students to take on?

What Does It Mean To Subtract

Mentally solve the following problem:

"Jake had 53¢ in his pocket. As he walked, 19¢ fell out of his pocket. How much money does Jake now have in his pocket?"

Share strategies whole group.

Thinking About Subtraction

- Work with a partner to explore the different methods for subtracting.
- Become familiar with each procedure by trying it out. Make-up more problems for you and your partner to try the different methods.
- Discuss with your partner why the method works and be able to explain it.

Comparing Methods

- How are Methods A and B equivalent? Why do they result in the same answer?
- Select two other methods and explain why they result in the same answer.
- Discuss with your group why the methods work.
- Whole group discussion.

Time of Reflection

Take a few moments to reflect on our time of thinking and learning today.

-- Jot down the meaningful and significant things you thought about.

-- Jot down the ways you thought mathematically and pedagogically.

-- Jot down how you contributed to our shared community of professionals.

Stay Safe

- Please help us put the room in proper order.
- Please leave your name tags for next time.